COLLABORATIVE JAMMING MITIGATION ON BLOCK-FADING CHANNELS

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***Abstract*— Collaborative techniques have been proposed to improve communications in the presence of a hostile jammer. A distributed detection and decoding technique based on collaborative decoding has been shown to be effective for communications over non-fading channels for coherent communications. In this paper, we consider collaborative communications strategies for jamming channels in which the message and jamming signals experience independent block fading at each receiving node. Furthermore, we assume that the phase of the message signal is not known and must be estimated from the received signal. Joint decoding and estimation of all of the fading and jamming parameters is computationally prohibitive, so we develop iterative estimation and decoding algorithms. The fading coefficient and parameters of the jamming signal are estimated using the EM and BCJR algorithms. The results show that the proposed technique is an effective response to hostile jamming.**

I. INTRODUCTION

We consider communication over block-fading channels, which are non-time selective over each packet, in the presence of partial-time jamming. Even in the absence of jamming, communication in block-fading channels requires relatively high average signal-to-noise ratios (SNRs). When jamming is present, accurate estimates of the fading coefficients are necessary to discriminate between jammed and unjammed symbols. Many algorithms have been presented for accurate estimation of the fading coefficients (see, for example, [1]–[4]) for time-selective fading. The situation becomes more difficult if a jammer is present. There has also been much research aimed at overcoming the jammer's effect (see for example, [5], [1], [6]) for non-fading channels. In this paper, we present collaborative techniques that can improve performance in the presence of block fading and jamming.

We consider a scenario in which a single transmitter communicates with multiple receiving nodes. The scenario that we envision is particularly applicable to communications between clusters in military ad hoc networks. In these systems, communications between clusters is over low-rate links (often tens of kpbs) that are particularly sensitive to jamming because of the low received signal power, and communication within a cluster may use much higher-speed wireless LAN technology (at several Mbps) that is less susceptible to jamming because of the higher received signal power within the cluster. Thus, we wish to use the nodes in the cluster to form a distributed antenna array to provide diversity against fading and to help mitigate the jamming signal. However, unlike a conventional antenna array, we do not assume that the phases of the oscillators at different receivers are synchronized or even known to each other. Nor do we make any assumption about the spacing of the nodes forming the array, other than that they are far enough separated to experience independent fading for both the communication and jamming signals. Thus, the usual beamforming techniques cannot be directly applied.

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The jamming signal is modeled by a Gaussian two-state hidden Markov model. To exploit the fact that each node receives the same message with independent noise, we have proposed collaborative jamming mitigation algorithms [7]. In our previous work, we show that these techniques are effective for nonfading channels with coherent detection (the phases of the message signals are assumed known). In this paper, we extend the jamming mitigation techniques to block-fading channels and address the problems of phase acquisition and channel estimation in the presence of jamming.

Detecting the message in the presence of many unknown parameters of the fading and jamming is a difficult problem. To reduce the complexity of the detection problem, we decompose it into a series of detection and estimation problems and use iteration to refine the estimates. One of the most important estimation problems is the estimation of the fading coefficients, for which we use the expectation-maximization (EM) algorithm [8], [9]. Given the estimates for the fading coefficients and jamming parameters, the joint-density approach to jamming mitigation [7] can be applied for block-fading channels. We analyze and simulate the performance of this scheme.

II. SYSTEM MODEL

As in [7], we consider the scenario in which a single transmitter is communicating with a distant cluster of N nodes in the presence of jamming. Each node receives independent copies of the same message and jamming signal. Both the message and jamming signals undergo block fading that is independent from each other and independent between nodes. The nodes in the cluster are assumed to be close enough to reliably exchange messages in the presence of the jamming signal. We consider a partial-time jammer that is modeled by a two-state hidden Markov model. The jamming signal is only present in state 1 (the ON state), and is modeled as white Gaussian noise with power spectral density (PSD) N_j/ρ. Here, ρ is the steady-state probability of being in the ON state. The expected time (in the number of channel symbols)

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in the ON state is $E\{T_j\}$. Gaussian thermal noise with power spectral density N_0 is present in both states 0 and 1. To provide diversity against jamming, the message is encoded by a turbo code, and the coded symbols are passed through a rectangular interleaver before transmission to break up jamming bursts at the input to the decoder. The overall system model is illustrated in Fig. 1. The received symbol at node i can be modeled as

$$y_k^{(i)} = a^{(i)}\sqrt{E_s}u_k + n_k^{(i)} + z_k b^{(i)} J_k, \tag{1}$$

where $a^{(i)}$ and $b^{(i)}$ are complex block-fading coefficients for the message and jamming signal, respectively. E_s is the symbol energy, and u_k is the message bit which is ± 1. The parameter z_k is an indicator random variable such that if $z_k = 1$, the kth bit is jammed and if $z_k = 0$, the bit is unjammed. Here, $n_k^{(i)}$ is complex Gaussian noise with variance N_0, and J_k is the jamming signal, which is a circularly symmetric complex Gaussian random variable with zero-mean and variance N_j/ρ. Therefore at the ith node, the total variance of the noise and jamming for state 1 is $N_0 + |b^{(i)}|^2 N_j/\rho$.

III. Fading Coefficient Estimation and Jamming Detection/Mitigation

Consider the problem of collaborative jamming mitigation in a group of N nodes. The problem is to accurately estimate the message u_k in the presence of unknown fading coefficients and partial-time jamming with unknown parameters. Suppose that each node has a copy of all of the received symbols from every other node. Then consider estimating the value of a particular message bit u_k given the vector of received symbols from all nodes, $\mathbf{Y} = [\mathbf{y^{(0)}}\ \mathbf{y^{(1)}} \cdots \mathbf{y^{(N-1)}}]$, where $\mathbf{y^{(i)}}$ is the received vector at the ith node, which can be represented as $\mathbf{y^{(i)}} = [y_0^{(i)}\ y_1^{(i)} \cdots]$. One approach is to find the value of u_k given by the joint MAP estimate for u_k, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{z}$, and $\mathbf{J}$,

$$\hat{u}_k = \operatorname{argmax} f(\mathbf{Y}|u_k, \mathbf{a}, \mathbf{b}, \mathbf{z}, \mathbf{J}) p(u_k) f(\mathbf{a}) f(\mathbf{z}) f(\mathbf{bJ}), \tag{2}$$

where the maximization is over $u_k \in \{+1, -1\}$, $z_k \in \{0, 1\}$, $a^{(i)}$, $b^{(i)}$, $J_k \in \mathbb{C}$ for all i, k, where $\mathbb{C}$ represents the set of complex numbers. All bold variables are similarly defined as $\mathbf{y^{(i)}}$. This is a difficult optimization problem, so as in previous work [1]–[4], iterative estimation is used to approximate the joint estimate. Given the fading coefficients $\mathbf{a}$ for the message, the jamming detection and mitigation algorithms in [7] can be applied with only small modifications. We begin as in [7] by assuming that the nodes exchange the received values for all of the jammed symbols. The overall detection and estimation procedure is illustrated in Fig. 1.

A. Estimation of the fading coefficients

The MAP estimator for the fading coefficient at node i is

$$\hat{a}_{\text{MAP}}^{(i)} = \underset{a^{(i)}}{\operatorname{argmax}}\ f(\mathbf{y^{(i)}}|a^{(i)}) f(a^{(i)}).$$

However, this is not practical because it requires complex and time-consuming numerical search. In this subsection, we apply the EM algorithm ([8], [9]) to estimate the fading coefficients $a^{(i)}$. We consider estimation of the fading coefficient a at one node, where we have dropped the superscript (i). The EM

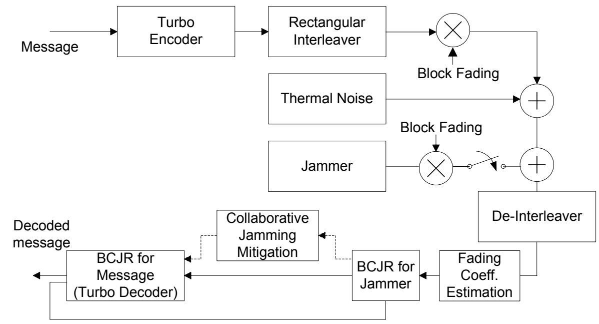

Fig. 1. The overall system model.

algorithm is a general estimation method when the observed data is incomplete or missing. In our derivation, the vector of jamming states $\mathbf{z}$ is considered as the "missing data". Then, the log-likelihood of the complete data can be written as

$$\begin{aligned} \log p(\mathbf{y}, \mathbf{z}|a, \mathbf{u}) &= \log(\mathbf{y}|a, \mathbf{z}, \mathbf{u}) p(\mathbf{z}|a, \mathbf{u}) \\ &= \sum_{k=0}^{N-1} \log p(y_k|z_k, u_k, a) \\ &\quad + \log \Big\{ p(z_0) \prod_{i=1}^{N-1} p(z_i|z_{i-1}) \Big\}. \end{aligned} \tag{3}$$

Let a' denote the estimate of a from the previous iteration. For the expectation step, we calculate

$$\begin{aligned} Q(a, a') &= \sum_{\mathbf{z}\in\mathcal{Z}} \log p(\mathbf{y}, \mathbf{z}|a, \mathbf{u}) p(\mathbf{z}|\mathbf{y}, a', \mathbf{u}) \\ &= \sum_{\mathbf{z}\in\mathcal{Z}} \log p(z_0) p(\mathbf{z}|\mathbf{y}, a', \mathbf{u}) \\ &\quad + \sum_{\mathbf{z}\in\mathcal{Z}} \Big\{ \sum_{i=1}^{N-1} \log p(z_i|z_{i-1}) \Big\} p(\mathbf{z}|\mathbf{y}, a', \mathbf{u}) \\ &\quad + \sum_{\mathbf{z}\in\mathcal{Z}} \Big\{ \sum_{k=0}^{N-1} \log p(y_k|z_k, u_k, a) \Big\} p(\mathbf{z}|\mathbf{y}, a', \mathbf{u}). \end{aligned} \tag{4}$$

We consider only the last term of (4) because the other terms do not depend on a. Therefore, we have

$$\begin{aligned} Q(a, a') &= \sum_{\mathbf{z}\in\mathcal{Z}} \sum_{k=0}^{N-1} \log \frac{1}{2\pi\sigma_i^2} \exp\Big\{ -\frac{|y_k - a\sqrt{E_s}u_k|^2}{2\sigma_i^2} \Big\} \\ &\qquad \cdot p(z_k|y_k, a', u_k) \\ &= \sum_{k=0}^{N-1} \sum_{i=0}^{1} \Big\{ C - \frac{|y_k - a\sqrt{E_s}u_k|^2}{2\sigma_i^2} \Big\} \Lambda_k(i) \Psi_k(i), \end{aligned} \tag{5}$$

where C does not depend on a. Here $\Lambda_k(i) = p(z_k = i, \mathbf{y}_0^k)$ and $\Psi_k(i) = P(\mathbf{y}_{k+1}^{N-1}|z_k = i)$ are the forward and backward jamming state probabilities, respectively, and can be calculated from the BCJR algorithm. The parameter σ_i^2 represents the variance in jamming state i and is generated in the BCJR algorithm as described in [10]. For the maximization step, the derivative of (5) with respective to a is calculated. Then we have

$$\hat{a}_{\text{EM}} = \frac{\sum_{k=0}^{N-1} \sum_{i=0}^{1} y_k u_k \Lambda_k(i) \Psi_k(i)/\sigma_i^2}{\sum_{k=0}^{N-1} \sum_{i=0}^{1} \Lambda_k(i) \Psi_k(i)/\sigma_i^2}, \tag{6}$$

to maximize (5). The estimate $\hat{a}_{\mathrm{EM}}$ is updated at each iteration.

Before starting the decoding process, we need an initial estimate of a. The accuracy of the initial estimate can have a great impact on the convergence of $\hat{a}_{\mathrm{EM}}$. We consider two approaches. In the first, a natural choice is the unbiased estimator

$$\hat{a}_{unbiased} = \frac{1}{|S|}\sum_{k\in S} y_k u_k$$

for a set S of known pilot symbols. The receiver uses the average amplitude of the pilot symbols as the initial estimate of a. Since some portion of the pilot symbols can be jammed, the estimate may be inaccurate because the variance of jammed symbols is much higher than that of unjammed symbols.

In the second approach, we use the two-dimensional histogram proposed in [10]. In this approach, a set of symbols that are likely to be unjammed is found without knowledge of the signal amplitude. This approach is based on the different variances of the jammed and unjammed signals. Since unjammed signals have a much lower variance than jammed symbols, it is likely that the unjammed symbols will be more closely clustered together. After generating a histogram for the received symbols, the bin with the largest number of symbols is most likely to contain the most unjammed symbols and fewest jammed symbols. The initial estimate of a is generated by averaging the symbols in that bin and then correcting for any phase ambiguity using pilot symbols. By choosing the size of the histogram bin appropriately, the variance of the histogram approach can be made much smaller than the simple, unbiased estimator based on using only pilot symbols. Although not shown because of space constraints, analysis indicates that a bin width equal to $1.6(2E_s/N_0)^{-1/2}$ approximately minimizes the variance of this estimator.

B. Jamming Detection/Mitigation

To perform jamming mitigation, each node must detect which symbols are jammed. Given the (estimated) fading coefficients, each node detects the jammer's state using an iterative MAP algorithm as in [11], [7]. To exploit the fact that the jamming signal is modeled by a Markov source, the BCJR MAP algorithm is used. During the detection process, the jammer's parameters, i.e. the variance of state 1 and the jammer's state transition probabilities, are estimated using the EM approach, which gives the same estimators described in [10]. The detection process depends on the estimated density of the received symbols given the estimated fading coefficients, for which the estimated values are used. Note that each node estimates the set of jammed symbols independently, so they use voting to determine a consensus set of jammed symbols [7]. Every node broadcasts the bit indices of the symbols that are estimated to be jammed. We use majority voting, which provides a trade-off between performance and overhead [7].

Based on the estimated set of jammed symbols and jamming parameters, the joint density technique is applied for jamming mitigation. The decoder uses the BCJR algorithm, which generates the *a posteriori* LLRs of the message bits by calculating the forward state metric $\alpha_k(s')$, the backward state metric $\beta_k(s)$, and the branch metric $\gamma_k(s', s)$. The two state metrics $\alpha_k(s')$, $\beta_k(s)$ are calculated in a recursive manner, and $\gamma_k(s', s)$ is calculated as

$$\gamma_k^{(i)}(s', s) = p(u_k)p(y_s^{(i)}|s', s)p(y_p^{(i)}|s', s), \tag{7}$$

where $y_s^{(i)}$ and $y_p^{(i)}$ are the matched filter outputs at the ith node for the kth systematic and parity bit, respectively. We can utilize the received symbols from every node by employing the joint density function for them. However, to do so would require exchanging the soft decisions for all of the received symbols. As the performance is often dominated by the jammed symbols, we assume that only the jammed symbols are exchanged. The joint density is used for calculating γ for these symbols.

It is straightforward to derive the covariance matrix between each node's received symbols. Let $\mathbf{y}_k = [y_k^{(0)}, y_k^{(1)}, \cdots, y_k^{(N-1)}]^T$. Then the mean vector of $\mathbf{y}_k$ is $\mathbf{m}_k = [\mu_k^{(0)}, \mu_k^{(1)}, \cdots, \mu_k^{(N-1)}]^T$, where $\mu_k^{(i)} = a^{(i)}\sqrt{E_s}u_k$. The covariance matrix Σ_k conditioned on $\mathbf{b}$, is defined as

$$\Sigma = E[(\mathbf{y}_k - \mathbf{m}_k)(\mathbf{y}_k - \mathbf{m}_k)^H],$$

where the diagonal elements are given by

$$\Sigma_{ll} = N_0 + |b^{(l)}|^2 N_j/\rho, \quad l = 0, \cdots, N-1.$$

Note that N_0 is assumed to be known, and $|b^{(i)}|^2 N_j/\rho$ is estimated using the forward-backward algorithm during the decoding process and updated after every iteration [7]. The upper off-diagonal elements are given by

$$\Sigma_{lm} = b^{(l)} b^{(m)*}\frac{N_j}{\rho},$$

where $l = 0, \cdots, N-1$, $m = l+1, \cdots, N-1$. The off-diagonal elements Σ_{lm} are estimated from the temporal average of the correlation between jammed symbols from the lth and mth nodes. The lower off-diagonal elements are conjugates of the upper off-diagonal elements. Then the joint density function of the received symbols is

$$f(\mathbf{y}_k|u_k, \mathbf{a}, \mathbf{b}) = \frac{1}{\pi^N|\Sigma_k|}\exp\left[-(\mathbf{y}_k - \mathbf{m}_k)^H \Sigma_k^{-1}(\mathbf{y}_k - \mathbf{m}_k)\right].$$

IV. Upper Bounds

In this section, we derive upper bounds for the performance of the joint pdf scheme. we use the approach proposed in [12] to bound the FER as

$$\mathrm{FER} \le \int_{a^{(0)}}\cdots\int_{a^{(N-1)}}\int_{b^{(0)}}\cdots\int_{b^{(N-1)}} \min\left\{1, \sum_{d=d_{free}}^{\infty} A_d P_d\right\} da^{(0)}\ldots da^{(N-1)} db^{(0)}\ldots db^{(N-1)}. \tag{8}$$

Here, P_d is the pairwise error probability for codewords at distance d given the fading coefficients for the message and jamming signals. The analytical expression for P_d is derived in the Appendix. Because of the min operation, P_d cannot be integrated over the fading statistics, and the number of integrations becomes computationally prohibitive. However, we can compute (8) for two special cases with $N = 2$. We show in the Appendix that for $N = 2$, P_d depends only on

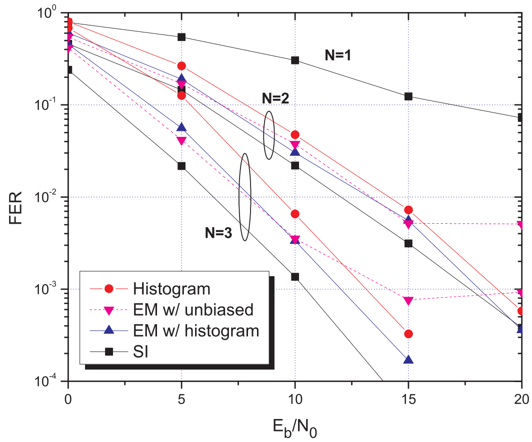

Fig. 2. Performance of the joint pdf scheme with estimated parameters.

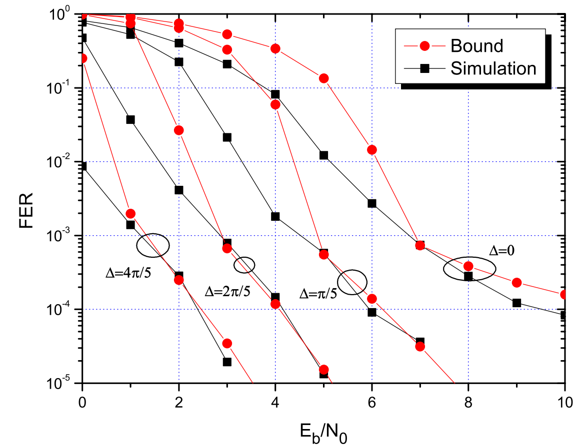

Fig. 3. Comparison of simulation and bounds for joint pdf scheme with nonfading message signals ($a^{(i)} = 1$ for all i), $E_b/N_j = 0$, $\rho = 0.6$.

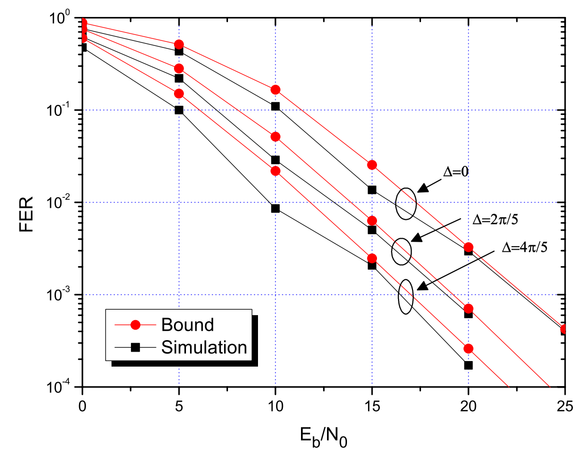

Fig. 4. Comparison of simulation and bounds for joint pdf scheme with nonfading jamming signals ($b^{(i)} = 1$ for all i), $E_b/N_j = 0$, $\rho = 0.6$.

the magnitudes of $a^{(i)}$, $b^{(i)}$, and the relative phase difference $\Delta \triangleq \angle a^{(1)} - \angle a^{(0)} + \angle b^{(1)} - \angle b^{(0)}$. By applying this, we can reduce the required number of real integrations from 8 to 5. We consider the special case of $a^{(i)} = 1$ for all i (nonfading message channels) and the case of $b^{(i)} = 1$ for all i (nonfading jamming channels), which each only require two numerical integrations. As shown in Section V, these result in tight bounds.

V. Results

In this section, we present several simulation results. Turbo codes with identical constituent codes with feedforward polynomial $1+D^2$ and feedback polynomial $1+D+D^2$ are used. The blocksize is 1000 information bits, and the code rate is 1/3. The number of total decoder iterations is limited to 10, and information is exchanged after the fifth iteration. The size of the rectangular interleaver is 55×55. For all of the results, $E_b/N_j = 0$ dB, and $\rho = 0.6$.

A. Jamming Detection/Mitigation

The results in Fig. 2 illustrate the performance of the joint pdf scheme with the jamming parameters, i.e. transition probabilities and jammer's variance, estimated via the forward-backward algorithm. The labels 'EM w/ unbiased' and 'EM w/ histogram' correspond to the case that the estimate of the fading coefficient is updated using the EM algorithm and the initial estimate is obtained from unbiased estimator and histogram, respectively. The label 'Histogram' corresponds to the case where the initial estimate obtained from histogram upon reception of the packet and used without update during the collaborative jamming mitigation process. For comparison, we include the performance with perfect channel state information (CSI). When we use the initial estimate obtained from histogram without update during the decoding process, we observe a moderate performance degradation. The performance degradation from this suboptimal estimate of $a^{(i)}$ increases as N increases. Updating the estimate using the EM algorithm produces a gain of approximated 0.5 dB and 1.0 dB for $N = 2$ and $N = 3$, respectively. We observe that the inaccurate initial estimate causes an error floor for high E_b/N_0. This shows that the resolution of the estimate is important, especially in the high SNR region. Without an accurate estimate of a, it is difficult to accurately estimate the jamming state, which causes the failure of the decoding process. Compared with the case of $N = 1$, a huge gain is obtained using the joint pdf scheme.

B. Upper Bounds for the Joint Density Scheme

Figs. 3 and 4 illustrate simulation and analytical results for the joint pdf scheme for the two special cases described in Section IV. For these results, we assume that the fading coefficients and jamming parameters are known to the receiver. The bounds are shown to be very tight. The overbound technique proposed in [13] is used to account for high weight codewords of the turbo codes.

VI. Conclusions

In this paper, we have investigated the application of collaborative jamming mitigation for block-fading channels with partial-time interference. The proposed jamming mitigation techniques use iterative detection, estimation, and decoding. The EM algorithm is used to estimate the fading coefficients, and we consider two different approaches to generating the initial estimate. Simulation and analytical results show that the joint pdf scheme is an effective jamming mitigation technique.

VII. Appendix

We derive P_d in block-fading channel assuming ML decoding is performed. The derivation is done assuming perfect side information for jamming state is available. For simplicity, assume $N = 2$ and the all-ones codeword (denoted by $\underline{1}$) is transmitted. If $\underline{c}$ is a competing codeword at distance d from the true codeword, the pairwise error probability is

$$P_d = p_E(\underline{c}) = p\Big(\log p(\underline{y}|\underline{1},\underline{z}) < \log p(\underline{y}|\underline{c},\underline{z})\Big),$$

where $\underline{z}$ is jammer's state sequence. We condition on d_1 symbols being jammed out of the d symbols that differ between $\underline{c}$ and the correct word. Thus, $d_0 = d - d_1$ symbols are unjammed. Let D_0 and D_1 be the set that contains the bit indices of the d_0 unjammed and d_1 jammed symbols, respectively. Then after some manipulations, we can write

$$\begin{aligned} p_E(\underline{c}) = p\Big(&\frac{2}{N_0}\sum_{i\in D0} T_i + 2C'_{00}\sum_{i\in D1} U_i + 2C'_{11}\sum_{i\in D1} V_i \\ &+2\sum_{i\in D1} W_i \;<\; 0\Big), \end{aligned} \quad (9)$$

where C'_{ij} is the ith row, jth column element of Σ^{-1}, $T_i = \mathrm{Re}(y_i^{(0)*}a^{(0)})$, $U_i = \mathrm{Re}(y_i^{(0)*}a^{(0)})$, $V_i = \mathrm{Re}(y_i^{(1)*}a^{(1)})$, and $W_i = \mathrm{Re}\left(\{y_i^{(0)*}a^{(1)} + y_i^{(1)}a^{(0)*}\}C'_{01}\right)$. Note that T_i and U_i have the same mean but different variance. It can be easily calculated that

$$\begin{aligned} E[T_i] &= |a^{(0)}|^2\sqrt{E_s}, \quad \mathrm{Var}[T_i] = |a^{(0)}|^2 N_0/2 \\ E[U_i] &= |a^{(0)}|^2\sqrt{E_s}, \quad \mathrm{Var}[U_i] = |a^{(0)}|^2\Sigma_{00}/2 \\ E[V_i] &= |a^{(1)}|^2\sqrt{E_s}, \quad \mathrm{Var}[V_i] = |a^{(1)}|^2\Sigma_{11}/2 \\ E[W_i] &= 2\sqrt{E_s}\,\mathrm{Re}\left(a^{(0)*}a^{(1)}C'_{01}\right) \\ \mathrm{Var}[W_i] &= 2|a^{(0)}|^2|C'_{01}|^2\Sigma_{11} + |a^{(1)}|^2|C'_{01}|^2\Sigma_{00}^2 \\ &\quad + \mathrm{Re}(a^{(0)*}a^{(1)}b^{(0)*}b^{(1)*}C'^2_{01})\frac{N_j}{\rho} \end{aligned}$$

Note that U_i, V_i, and W_i are not independent of each other. So we calculate the covariance between them as

$$\begin{aligned} \mathrm{Cov}(U_i,V_i) &= E(U_iV_i) - E(U_i)E(V_i) \\ &= \mathrm{Re}(a^{(0)*}b^{(0)}a^{(1)}b^{(1)*})\frac{N_j}{2\rho} \\ \mathrm{Cov}(U_i,W_i) &= \mathrm{Re}(a^{(0)*}a^{(1)}C'_{01})\Sigma_{00}/2 \\ &\quad +|a^{(0)}|^2\,\mathrm{Re}(b^{(0)*}b^{(1)}C'_{01})\frac{N_j}{2\rho} \\ \mathrm{Cov}(V_i,W_i) &= \mathrm{Re}(a^{(0)*}a^{(1)}C'_{01})\Sigma_{11}/2 \\ &\quad +|a^{(1)}|^2\,\mathrm{Re}(b^{(0)*}b^{(1)}C'_{01})\frac{N_j}{2\rho}. \end{aligned}$$

The terms in (9) except the first summation can be represented as

$$\sum_{i\in D1} Y_i$$

where

$$Y_i \triangleq 2C'_{00}U_i + 2C'_{11}V_i + 2W_i.$$

Define Υ be the argument in (9), then we can obtain

$$\begin{aligned} E[\Upsilon] &= \frac{2(d-d_1)|a^{(0)}|^2\sqrt{E_s}}{N_0} + d_1E[Y_i] \\ \mathrm{Var}[\Upsilon] &= \frac{2(d-d_1)|a^{(0)}|^2}{N_0} + d_1\,\mathrm{Var}[Y_i]. \end{aligned}$$

So, the pairwise error probability P_d conditioned on $\mathbf{a}$ and $\mathbf{b}$ is

$$P_d = \sum_{d_1=0}^{d} Q\left(\frac{E[\Upsilon]}{\sqrt{\mathrm{Var}[\Upsilon]}}\right)\binom{d}{d_1}\rho^{d_1}(1-\rho)^{d-d_1}.$$

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